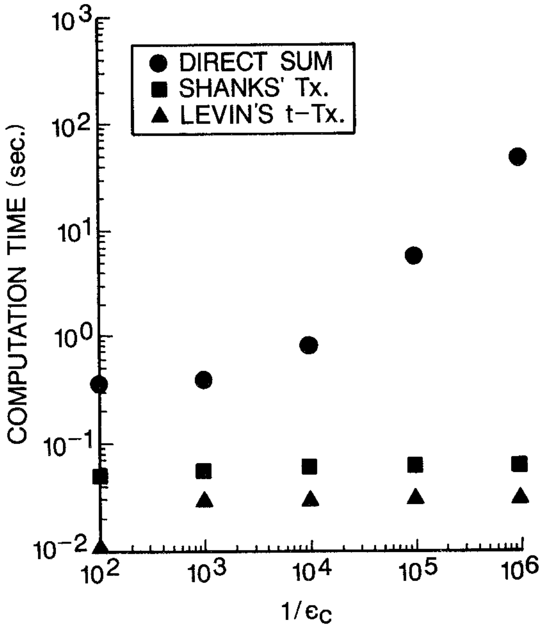


Fig. 2. Computation time versus $1/\epsilon_c$ for the spatial domain Green's function series in (4) for $(x, y) = (0.0\lambda, 0.6\lambda), d = 1.2\lambda$, and $\lambda = 1.0m$.

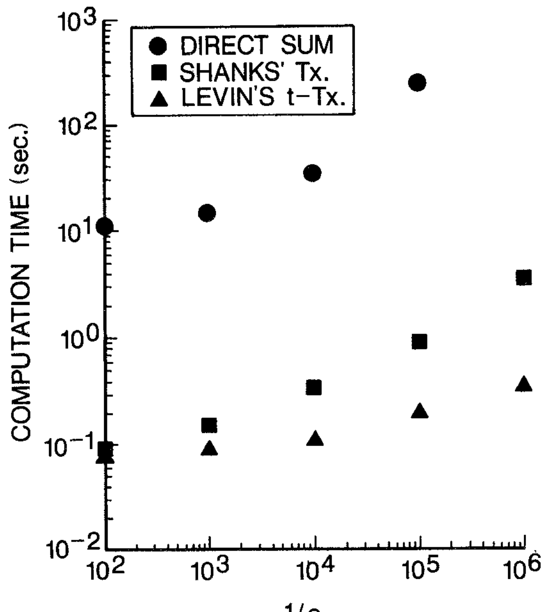


Fig. 3. Computation time versus $1/\epsilon_c$ for the spatial domain Green's function series in (6) for $(x, y, z) = (0.6\lambda, 0.6\lambda, 0.0\lambda), D_x = D_y = 0.75\lambda$, $p_0 = q_0 = 0, \lambda = 1.0m$.

have the slowest convergence. This results in a considerable savings in computation time.

References

[1] S. Singh, W. F. Richards, J. R. Zinecker, and D. R. Wilton, "Accelerating the convergence of series representing the free space periodic Green's function," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 1958–1962, Dec. 1990.

[2] R. E. Jorgenson and R. Mittra, "Efficient calculation of the free-space periodic Green's function," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 633–642, May 1990.

[3] S. Singh and R. Singh, "On the use of Shanks's transform to accelerate the summation of slowly converging series," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 608–610, Mar. 1991.

[4] D. Levin, "Development of non-linear transformations for improving convergence of sequences," *Int. J. Computer Math.*, Sec. B, vol. 3, pp. 371–388, 1973.

[5] D. Shanks, "Non-linear transformations of divergent and slowly convergent sequences," *J. Math. Physics*, vol. 34, pp. 1–42, 1955.

[6] P. Wynn, "On the convergence and stability of the epsilon algorithm," *SIAM J. Numer. Anal.*, vol. 3, pp. 91–122, 1966.

Computer-Aided Design of a Singly-Matched (S-M) Multiplexer with a Common Junction

Ji-An Gong and Wai-Kai Chen

***Abstract*—In this paper, the formulas based on scattering parameters are presented for the design of a multiplexer composed of $n - 1$ channel equalizers connected either in parallel or in series at a common junction with a 1-Ω resistive generator and $n - 1$ channel complex loads. It is known as the singly-matched (S-M) multiplexer. A new two-stage computer-aided design approach is developed for the S-M multiplexer. A design example of a three-channel singly-matched multiplexer including the designs of three individual S-M channel equalizers is given to demonstrate the approach.**

I. Introduction

Many multiplexer design techniques have been developed [1]–[3], but none of them considers the complex load impedances. With recent developments in solid-state technology, a pure resistive model of the load is no longer an adequate representation.

In this paper, a multiplexer configuration consists of $n - 1$ channel equalizers connected in either parallel or series at a common junction. All the loads of the multiplexer are assumed to be complex, and the generator is assumed to be connected in series with a 1-Ω resistor. A multiplexer with a resistive source and complex loads is called a singly-matched (S-M) multiplexer. Formulas are presented for either the parallel or series configuration with a common junction. A new two-stage computer-aided procedure is developed for their design. At the first stage, each channel equalizer is designed to be a singly-matched so that the transfer of power from the 1-Ω resistive generator to the channel complex load is maximized over a prescribed channel frequency band. This is known as a single broad-band matching problem for which many papers have been published [4]–[6]. Since the ladder structure is attractive not only from a practical viewpoint, but also effective as an equalizer in most applications, in this paper each channel equalizer is assumed to be a two-port lossless ladder network. The S-M channel equalizer is realized by optimization matching technique, thereby making it easier to design a S-M equalizer having different types of responses (Chebyshev or elliptic) and various passbands (lowpass, bandpass, or highpass). At the second stage, by using the formulas and existing optimization techniques, all the element values in the multiplexer are modified until a good match is achieved at the common input port over the entire transmission band. Since all the designs are

0018–9480/93$03.00 © 1993 IEEE

accomplished by computer optimization, modeling of the channel complex load is not needed. Finally, a design example of a three-channel S-M multiplexer including the designs of three individual S-M channel equalizers is given to demonstrate the approach.

II. Formulas

Consider a S-M multiplexer composed of $n-1$ channel equalizers connected in parallel at a common junction, as shown in Fig. 1(a). If each channel equalizer E_i $(i = 2, 3, \cdots, n)$ is a lossless, reciprocal two-port network whose scattering matrix normalized to the 1- Ω source resistance and its channel complex loads Z_i $(i = 2, 3, \cdots, n)$ is

$$S_i = \begin{bmatrix} s_{11i} & s_{12i} \\ s_{21i} & s_{22i} \end{bmatrix}, \quad (1)$$

then the scattering matrix of the multiplexer normalized to the 1-Ω source resistance and $n-1$ complex loads Z_i $(i = 2, 3, \cdots, n)$ and its scattering parameters are found to be

$$S_p = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}, \quad (2)$$

$$s_{11} = \frac{\prod_{i=2}^{n}(1+s_{11i}) - \sum_{j=2}^{n}\left\{(1-s_{11j})\prod_{\substack{l=2\\ l\neq j}}^{n}(1+s_{11l})\right\}}{\hat{G}_p}, \quad (3)$$

$$s_{1k} = \frac{2s_{12k}\prod_{\substack{i=2\\ i\neq k}}^{n}(1+s_{11i})}{\hat{G}_p}, \quad (4)$$

$$s_{jk} = \frac{2s_{12k}s_{12j}\prod_{\substack{i=2\\ i\neq k,j}}^{n}(1+s_{11i})}{\hat{G}_p}, \quad (5)$$

$$s_{kk} = s_{22k} - \frac{(s_{12k})^2\sum_{\substack{i=2\\ i\neq k}}^{n}\left\{(1-s_{11i})\prod_{\substack{l=2\\ l\neq i,k}}^{n}(1+s_{11l})\right\}}{\hat{G}_p}, \quad (6)$$

$$\hat{G}_p = \prod_{i=2}^{n}(1+s_{11i}) + \sum_{j=2}^{n}\left\{(1-s_{11j})\prod_{\substack{l=2\\ l\neq j}}^{n}(1+s_{11l})\right\}. \quad (7)$$

For a S-M multiplexer composed of $n-1$ channel equalizer connected in series at a common junction, as shown in Fig. 1(b), there is a dual set of formulas that are the same as those for the parallel case except that all the reflection parameters s_{11i}, s_{22i} $(i = 2, 3, \cdots, n)$, and s_{kk} $(k = 1, 2, \cdots, n)$ are replaced by $-s_{11i}, -s_{22i}$, and $-s_{kk}$, respectively. The formulas may be proved in the same way as those in Section III of [3] except that we use the complex normalization. From these formulas it is seen that if we make s_{1k} as close as possible to s_{12k} over the kth channel passband, then for a parallel-connected S-M multiplexer, all s_{11i} $(i = 2, \cdots, n)$ except for s_{11k} must approach +1; whereas, for a series-connected one, they must approach -1, resulting in the following design approach.

III. Design Approach

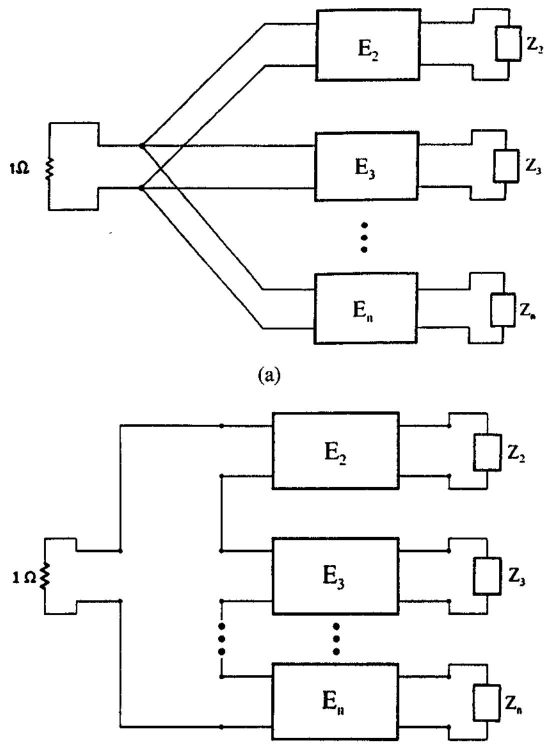


Fig. 1. (a) Parallel-connected S-M multiplexer. (b) Series-connected S-M multiplexer.

A. S-M Channel Equalizer Design

From given specifications, the response type, degree, and the ripple factor of each channel equalizer E_k are determined, and its two-port lossless ladder configuration is selected. Since the equalizer E_k is lossless, $S_k(j\omega)$ is unitary. Hence, the transducer power gain of the equalizer E_k can be written as

$$T_k(\omega^2) = 1 - |s_{11k}(j\omega)|^2 \quad (8)$$

where the reflection coefficient s_{11k} [5] is

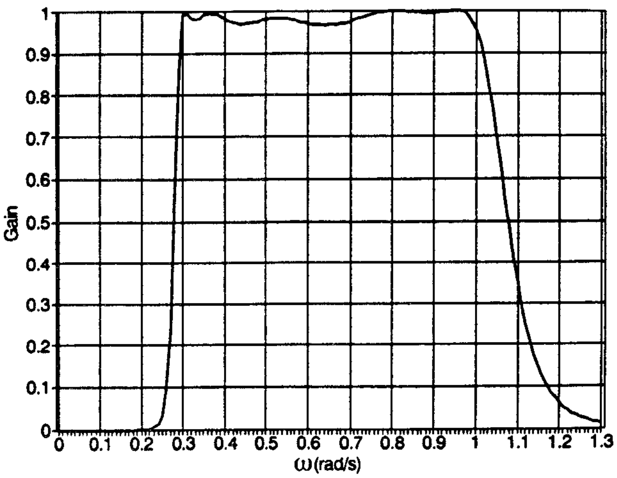
$$s_{11k}(j\omega) = (-1)^m \frac{g_{ok}(-j\omega)}{g_{ok}(j\omega)} \frac{Z_k(j\omega) - Z_{ok}(-j\omega)}{Z_k(j\omega) + Z_{ok}(j\omega)} \quad (9)$$

where $m = 0$ or $m > 0$ for low-pass or band-pass matching networks,

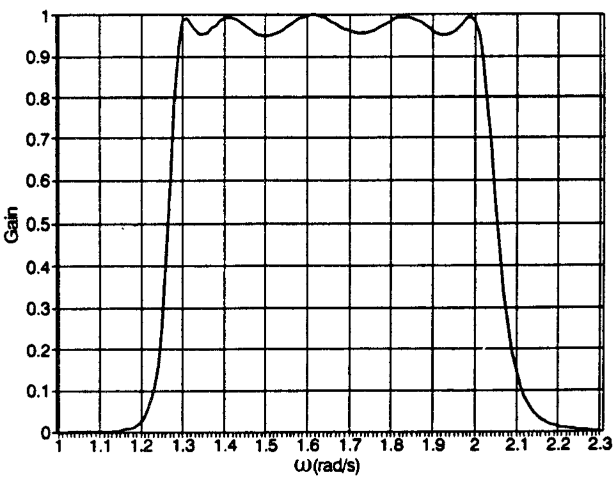
$$Z_{ok}(j\omega) = \frac{h_{ok}(j\omega)}{g_{ok}(j\omega)} \quad (10)$$

which is the driving-point impedance of the equalizer at the output port and can be obtained from the selected channel equalizer configuration, and $Z_k(j\omega)$ may be a set of calculated values obtained from the model of the kth channel load or a set of measured values from its device.

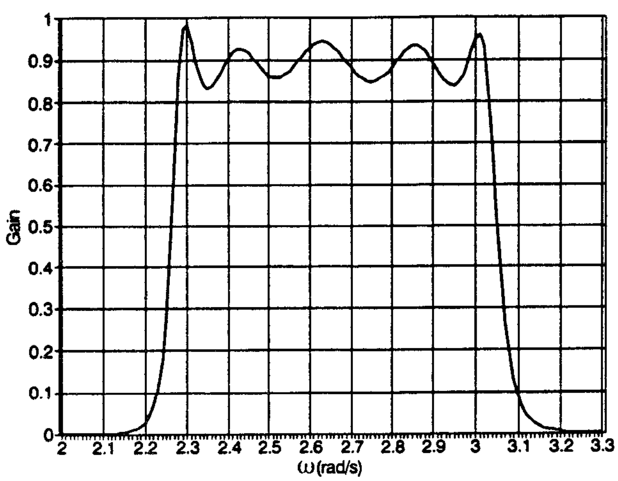
The design of the S-M channel equalizer E_k is realized by using (9) and a new optimization matching technique in which all the element values of E_k are taken to be the optimization variables. In order to obtain the initial values of the optimization variables, a filter terminated in a 1-Ω resistor instead of Z_k is designed over the kth channel frequency band, and the resulting element values are taken as the initial values of the design of E_k. In the optimization process, $|s_{11k}(j\omega)|$ is minimized so that $T_k(\omega^2)$ is optimized over the kth channel frequency band. Next three design examples show that it is a simple effective approach to single broad-band matching.



(a)

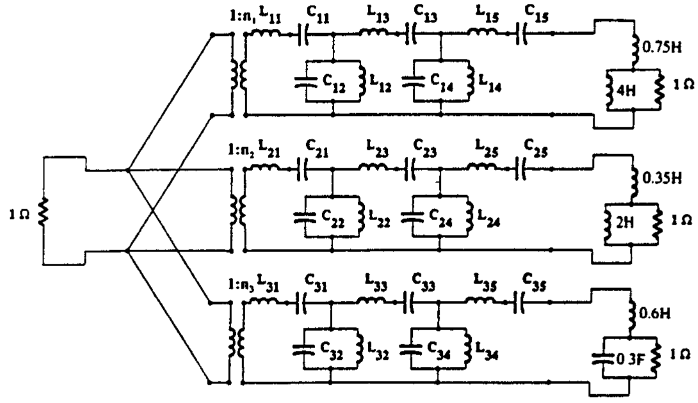


(b)

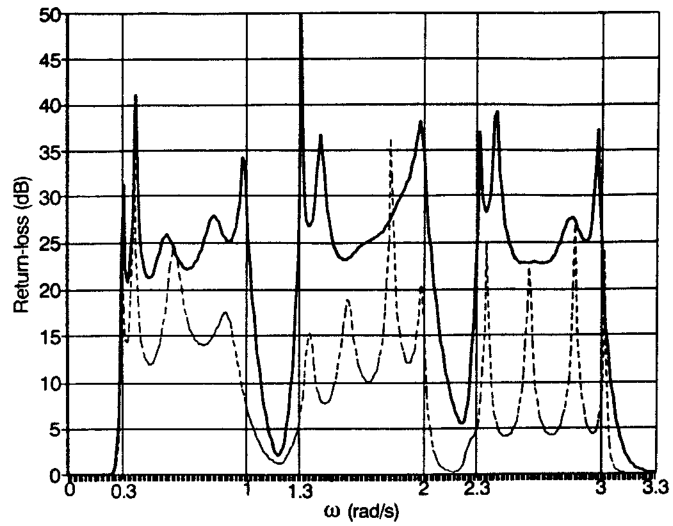


(c)

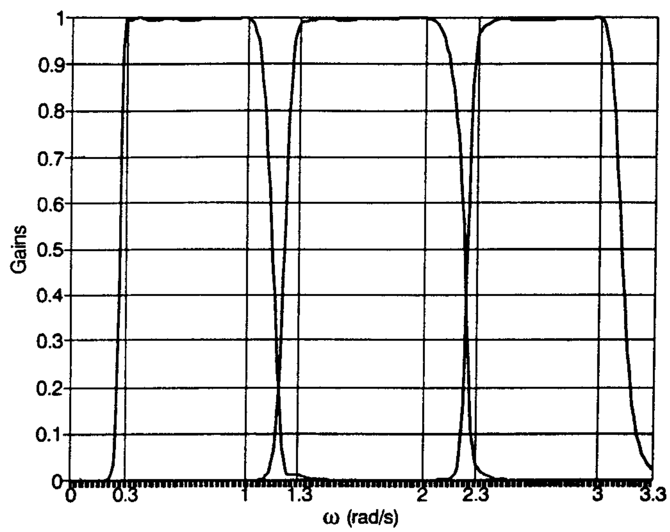
Fig. 2. Gain responses of individual channel S-M equalizers. (a) Channel-1. (b) Channel-2. (c) Channel-3.



(a)



(b)



(c)

Fig. 3. (a) Three-channel S-M multiplexer with three complex loads. (b) Its return-loss characteristics before and after the optimization (dashed line and solid line). (c) Its three gain responses $|s_{21}|^2, |s_{31}|^2$, and $|s_{41}|^2$ at three channel output ports.

B. S-M Multiplexer Design

The desired single-matched multiplexer consists of $n - 1$ S-M channel equalizers connected in either parallel or series at a common input port, as shown in Fig. 1(a) or 1(b). The design is accomplished by computer optimization using the multiplexer formulas presented above. The objective function F of the optimization may be expressed as

$$F = \frac{1}{m} \sum_{j=1}^{m} w_j |s_{11}(\omega_j)|^k \tag{11}$$

TABLE I
THREE CHANNEL S-M MULTIPLEXER ELEMENT VALUES

Channel 1	L_{11}	C_{11}	L_{12}	C_{12}	L_{13}	C_{13}	L_{14}	C_{14}	L_{15}	C_{15}	n_1
Initial values	1.45588	2.33876	1.70569	1.94551	2.70717	1.24432	1.49288	2.12455	0.57708	1.69317	1.02739
Optimized values	1.64496	2.98511	1.49597	2.29051	2.06776	1.54998	1.37163	2.07975	0.00012	2.12026	0.99235
Channel 2	L_{21}	C_{21}	L_{22}	C_{22}	L_{23}	C_{23}	L_{24}	C_{24}	L_{25}	C_{25}	n_2
Initial values	1.59245	0.24215	0.19715	1.95495	2.84822	0.13483	0.19610	1.95948	1.58387	0.18245	0.98847
Optimized values	1.47558	0.25207	0.20774	1.78798	2.01456	0.18447	0.20582	1.80031	0.65031	0.31748	1.01553
Channel 3	L_{31}	C_{31}	L_{32}	C_{32}	L_{33}	C_{33}	L_{34}	C_{34}	L_{35}	C_{35}	n_3
Initial values	1.63817	0.08824	0.07364	1.95882	2.82151	0.05135	0.07428	1.95897	1.63721	0.07008	1.00005
Optimized values	0.51017	0.18749	0.05137	2.64548	1.53795	0.09257	0.04964	2.93161	0.30861	0.20218	1.03459

TABLE II
GAIN RESPONSE PARAMETERS OF FIG. 2

	G_{min}	ε
Channel 1	0.952	0.050
Channel 2	0.949	0.052
Channel 3	0.832	0.179

TABLE III
GAIN RESPONSE PARAMETERS OF FIG. 3(c)

	G_{min}	ε
Channel 1	0.993	0.007
Channel 2	0.988	0.012
Channel 3	0.966	0.035

where the w_j's are the weighting coefficients, ω_j $(j = 1, 2, \cdots, m)$ are the optimal sample frequency points, k is an even integer, and $s_{11}(\omega)$ is the reflection coefficient of the multiplexer. All the element values of the M-S multiplexer are taken to be the optimization variables, and the resulting element values in the design of the $n-1$ individual S-M channel equalizers are taken to be their initial values. During the optimization process, a minimization algorithm is used so that all the element values are modified in order to minimize the value of $|s_{11}(\omega)|$ until a good match is obtained at the common input port over the entire transmission frequency band.

From the above, the design procedure of the S-M multiplexer can be summarized as follows:

1. Determination of the response type, degree, and the ripple factor of each channel equalizer according to the specifications of the desired S-M multiplexer.
2. Determination of the two-port LC-ladder configuration of each channel equalizer according to the response type, degree, passband range, and the multiplexer connection type.
3. Design individual channel filters terminated in a 1-Ω resistor. The resulting element values are taken to be the initial values in the design of the individual S-M channel equalizer.
4. Single-match individual channel equalizers between the 1-Ω source resistance and the channel complex load over its passband by optimization using (11) and (9). The resulting element values are taken as the initial values of the S-M multiplexer design.
5. Single-match the multiplexer over the entire transmission frequency band by optimization using (11) and (3).

Although there are limits on achievable gains imposed by the load impedances [6], from the following designs it is seen that computer optimization design techniques can obtain excellent results close to the maximum achievable ones.

IV. ILLUSTRATIVE EXAMPLE

A three-channel singly-matched multiplexer has been designed, as shown in Fig. 3(a). The three channel bands are 0.3–1 rad/s, 1.3–2 rad/s, and 2.3–3 rad/s, respectively.

At first, a fifth-order Chebyshev bandpass ladder configuration is selected for each channel equalizer. Three Chebyshev bandpass filters terminated in a 1-Ω resistor are designed over the corresponding channel frequency bands, and their resulting element values are taken to be the initial values of optimization for the three individual S-M channel equalizers. Next, three individual S-M equalizers are designed by using the simple single-matching optimization technique in which 11 element values of the channel equalizer are taken to be optimization variables. We use the least squares minimization algorithm in the designs of the channel-1 and channel-2 S-M equalizers, and the DFP (Davidon-Fletcher-Powell) minimization algorithm in the design of the channel-3 S-M equalizer. Their optimized element values are listed as the initial values of the multiplexer elements in Table I. The gain responses of the three individual S-M channel equalizers are shown in Fig. 2, and their minimum passband gains G_{min} and ripple factors $\varepsilon, G_{max}/G_{min} - 1$, are listed in Table II.

Having finished the designs of the three individual S-M channel equalizers, we proceed with the design of the three-channel S-M multiplexer (Fig. 3(a)), the transmission characteristic of which is such that the transfer of power from the 1-Ω resistive generator to each channel load impedance over its channel passband will be maximized. All the design is carried out by a computer program SMDP (Single-Matched Multiplexer Design Program), which is written in Fortran language and by using the formulas and the DFP algorithm. In the process, all 33 element values in the multiplexer are taken to be the optimization variables. The weighting coefficients are taken to be $w_j = 22$, and $k = 4$. The 43 sample frequencies are taken over the passbands, and the user time and system time taken to perform the optimization on a Sun SLC 4/20 were 144.3 s and 0.4 s, respectively. Optimized element values of the three-channel S-M multiplexer are listed in Table I. The return-loss characteristics of the multiplexer before and after the optimization (dashed line and solid line) are shown in Fig. 3(b), giving a return-loss of the designed S-M multiplexer more than 20 dB over the three channel passbands. In other words, the reflected power of the S-M multiplexer at the input port is less than 1% of the total effective power over the channel passbands.

It is significant to note that the design results are excellent at the three channel output ports as shown in Fig. 3(c). The minimums of the three channel passband gains of the S-M multiplexer and their ripple factors are listed in Table III. In fact, the gains at most frequencies over the three channel passbands are in excess of 0.99. Furthermore, the gain responses of the three channels are greatly improved as compared with those of the individual channels (Fig. 2 and Table II), showing mutual compensations among channel equalizers.

From the designs it is found that in order to obtain a good result, it is important to choose appropriate sample frequencies in (11). The sample frequencies usually include the perfect transmission points, the side-points of channel passbands, and the lowest points at which the return-loss response has minimums. During the optimization process, it is often needed to make adjustment and even add new ones (the lowest points).

V. Conclusion

Formulas were presented for the design of a singly-matched multiplexer with a common junction. A new general design approach was developed by computer optimization using these formulas. A design example of a three-channel S-M multiplexer demonstrated this approach.

References

[1] J. D. Rhodes and S. A. Alseyab, "A design procedure for bandpass channel multiplexers connected at a common junction," *IEEE Trans. Microwave Theory Tech.*, vol. 28, pp. 246–253, 1980.

[2] M. H. Chen, F. Assal, and C. Mahle, "A contiguous band multiplexer," *Comsat Tech. Rev.*, vol. 6, pp. 285–305, 1976.

[3] J. A. Gong and W. K. Chen, "On the design of a multiplexer with a common junction," *Circ. Syst. Signal Process.*, vol. 9, pp. 75–97, 1990.

[4] W. K. Chen and K. G. Kourounis, "Explicit formulas for the synthesis of optimum broadband impedance matching networks II," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 609–620, 1978.

[5] H. J. Carlin and B. S. Yarman, "The double matching problem: analytic and real frequency solutions," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 15–28, 1983.

[6] W. K. Chen, *Theory and Design of Broadband Matching Networks*, Pergamon Press, New York, 1976.

Negative Differential Resistance (NDR) Frequency Conversion with Gain

R. J. Hwu, A. Djuandi, and S. C. Lee

***Abstract*—The dependence of the conductance of negative differential resistance (NDR) devices in the presence of an rf signal of varying amplitude has been theoretically analyzed. Variable absolute negative conductance has been observed in both unbiased resonant tunneling devices and biased tunnel diode when the applied pump power is within the correct range. The theoretical observation of the dependence of the dc conductance of the NDR devices on the power level of the applied pump signal is supported by the experimental results. Absolute negative conductance of NDR devices provides the possibility of oscillation and harmonic oscillation up to the cut-off frequency of the device. Biased oscillators and self-oscillating frequency multipliers have been experimentally demonstrated using a tunnel diode. Unbiased oscillators have also been successfully realized with two back-to-back connected tunnel diodes which exhibit an anti-symmetrical I-V characteristic.**

Manuscript received January 2, 1992; revised September 4, 1992. This work was supported by National Science Foundation Grant No. ECS-9110697.

The authors are with the Department of Electrical Engineering, University of Utah, Salt Lake City, UT 84112.

IEEE Log Number 9207432.

I. Introduction

There has been increased interest in the study of resonant tunneling devices as the nonlinear characteristics of these devices can be used for frequency conversion applications such as multipliers [1]–[3] and mixers [1], [2]. In addition, their ability to exhibit negative differential resistance (NDR) regions leads to their potential use as gain elements and presents new opportunities for circuit design [1], [2]. This paper will address the pros and cons of using such a potential gain element for frequency conversion applications.

A nonlinear-circuit analysis computer program has been developed to analyze the behavior of negative conductances of NDR devices. The analysis technique was developed by Kerr et al. [4] and the program was implemented for analyzing ideal Schottky barrier diodes by Siegel et al. [5]. The analysis program has been modified to take into account the negative resistance of an NDR device [6]. Since the devices were mounted on a 50 Ω microstrip line for the measurements in this work, an embedding impedance of 50 Ω at every harmonic frequency has been used. A simple experiment was carried out to verify that the embedding impedance at higher harmonic frequencies was indeed 50 Ω [7].

II. Absolute Negative Conductance

1. Nonlinear-Circuit Analysis Results

The I-V characteristics measured from a tunnel diode shown in Fig. 1(a) have been used in the nonlinear-circuit analysis. The expression for the capacitance of a junction diode was used in the analysis. The conductance of the tunnel diode biased at the center of the NDR region was studied. From the results shown in Fig. 1(b), an absolute negative conductance has been observed from the tunnel diode biased in the NDR region when the applied pump amplitude is limited to the appropriate range. The magnitude of the absolute negative conductance changes with the pumping amplitude. The pump amplitude required to achieve absolute negative conductance increases with increasing pump frequency. This can be easily explained using the equivalent circuit model of a NDR device. Since the impedance of the parallel circuit section decreases with increasing frequency, more of the applied voltage is distributed across the series resistance and less appears across the parallel circuit section for higher pump frequencies. The fact that an absolute negative conductance occurs for a tunnel diode biased in the NDR region implies that oscillation can occur at any frequency if the pumping power is within the region that negative conductance occurs.

2. Experimental Results

The measurement of the I-V characteristic was performed using a standard semiconductor analyzer. The rf signal was supplied by a HP signal generator and separated from the dc bias voltage using a standard bias network. During the experiment, it was established that the dc conductance of the NDR device is, indeed, highly dependent on the rf input signal. The I-V characteristics of a tunnel diode measured at different rf input power levels are shown in Fig. 2(a). The dependence of the dc conductance of the NDR device on the frequency of the rf input signal also observed during this measurement can easily be seen from the equivalent circuit model of the NDR device. It was also observed that the frequency dependence of the dc conductances is stronger for larger rf pump level. The I-V curves measured at different pump power levels and frequencies compare favorably to